Self-organization processes in laser system with nonlinear absorber and external force influence

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We discuss mechanisms of self-organization processes in two-level solid-state class-B laser system. The model is considered under assumptions of influence of nonlinear absorber and external force, separately. It was found that self-organization occurs through the Hopf bifurcation and results to a stable pulse radiation. Analysis is performed according to the Floquet exponent investigation. It was found that influence of the nonlinear absorber extends the domain of control parameters that manage a stable periodic radiation processes. An external force suppresses self-organization processes. A combined influence of both external force and nonlinear absorber results to more complicated picture of self-organization with two reentrant Hopf bifurcations.

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I. INTRODUCTION

The most intriguing phenomena in systems with nonlinear dynamics is a transition to the regime with dissipative structures formation. A related problem of such effects investigation in systems with large numbers of freedom degrees attracts an increasing attention in last three decades. Due to self-organization effects a number of freedom degrees is reduced and description of the system dynamics can be performed in terms of macroscopic variables. A typical picture is realized in laser systems where description is provided with a help of amplitudes of electric field (or intensity of the radiation), polarization and population inversion [1].

Laser theory shows that corresponding dissipative structures define formation of pulse or modulated signals in homogeneous systems or spirals in spatially extended ones [2]. In practice a formation of stable periodic radiation can be induced by introducing an additional medium with nonlinear properties which are realized as an absorber or modulator. Such type of lasing is known as passive one. Usually, such a kind of medium leads to nonlinear dependence of the relaxation time of the electric field amplitude

[3, 4] or nonlinear dependence of refractive exponent [5–8], composite material can be used to introduce different type of such nonlinearity [9–14]. A coherent dynamics of two-level laser systems in the presence of dispersive and absorptive effects was observed theoretically and experimentally [15, 16]. Statistical properties of self-organization effects of such type systems was discussed in [17]. Regimes of optical parametric oscillation in a semiconductor microcavity are studied in [18]. It was found that stationary behaviour of polarization can be described by the formalism of non-equilibrium transitions, where bistability is observed (see [5, 19]). It was shown that an oscillating lasing is realized inside a bounded domain of the system parameters. Another (active) way to initiate a coherent lasing is an introducing an external influence on nonlinear processes in the cavity [20]. An actual problem in laser physics is to find possible mechanisms and to set a range of control parameters that manage properties of stable periodic radiation (see for instance [3, 4] and citations therein). Despite this problem is still opened in deterministic (regular) systems a lot of attention is paid to find coherent regimes under influence of stochastic sources [21–24].

In this Paper, we are aimed to investigate the dynamics of the solid-state class-B laser systems which are simple in realization and are wide used in physical applications. We consider

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deterministic models only. According to the theoretical approach, based on Floquet analysis, we will explore in what a manner a nonlinear medium can induce a stable periodic radiation. It will be shown that varying in a saturation amplitude of electric field and absorption coefficient one can arrive at stable and unstable dissipative structures. Properties of self-organization process induced by an external force influence will be considered. At last, a combined influence of both nonlinear medium and external force on the system dynamics will be described.

The paper is organized in the following manner. In Section II we present a model of our system where we introduce theoretical constructions to model an influence of both an absorber and external force. Section III is devoted to development of the analytical approach to study process of dissipative structures formation. In Section IV we apply the derived formalism to investigate properties of stable periodic radiation in the presence of the absorber, external force and its combined affect. Main results and perspectives are collected in the Conclusion (Section V).

II. MODEL

Considering a prototype model for a two-level laser system, one deals with dimensionless variables such as: an electric field amplitude E, polarization P and S to be a population inversion. A standard technique usage [3] allows to reproduce evolution equations for these three macroscopic freedom degrees from both the Maxwell-type equation for electro-magnetic field and density matrix evolution equation. It leads to the system of Maxwell-Bloch type that is reduced to the Lorenz-Haken model in the form

$$\begin{cases} \varkappa^{-1}\dot{E} = -E + P, \\ \gamma_\perp^{-1}\dot{P} = -P + ES, \\ \gamma_\parallel^{-1}\dot{S} = (S_e - S) - EP. \end{cases} \quad (1)$$

For the single mode laser system a relaxation of electric field amplitude E is addressed to losses

in a bulk of the medium and characterized by the velocity $\varkappa = 1/2\tau_c$, where τ_c is a life-time of a photon in a cavity. $\gamma_\perp$ is a relaxation velocity of nondiagonal elements of density matrix which is related to the half-width of a spectral line. The relaxation scale for the population inversion is determined by the velocity $\gamma_\parallel$ defined by both transition probability between two energy levels and a corresponding frequency. S_e controls the pump intensity, as usual. The model (1) shows a linear combination of the amplitude E and polarization P, despite the evolution of both P and the pump intensity S are nonlinear. It is principally that the positive feedback of E and S leads to instability in the polarization that induces a self-organization. According to the Le-Shatelier principle such positive feedback is compensated through negative one in third equation (the last term).

To make an analysis we pass to dimensionless variables $\tau \equiv t\varkappa$, $\sigma \equiv \varkappa/\gamma_\perp$ and $\varepsilon \equiv \varkappa/\gamma_\parallel$. Hence, the system (1) takes the form

$$\begin{cases} \dot{E} = -E + P, \\ \sigma\dot{P} = -P + ES, \\ \varepsilon\dot{S} = (S_e - S) - EP. \end{cases} \quad (2)$$

Assuming different combinations between relaxation scales σ and ε, one can describe three possible classes of laser systems. At $\varepsilon, \sigma \ll 1$ we arrive at the laser models of class-A (organic dye lasers) with one-dimensional phase space, where systems states are represented by fixed points only. Here self-organization effects are described by a formalism of non-equilibrium phase transitions. Class-B (solid-state lasers) is characterized by a condition $\sigma \ll 1$. Here phase space is two-dimensional and transition processes are of oscillation type and hence self-organization processes result in dissipative structures formation. For the class-C (molecular gas lasers) we set $\sigma, \epsilon \sim 1$ and in three-dimensional phase space a strange attractor can be realized. At last, the class-D (beam masers) is characterized by condition $\sigma, \epsilon \gg 1$. In this Paper we consider the class-B only, where the polarization P is assumed to be a microscopic quantity and should be treated as fast variable which follows the electric field

amplitude E evolution. Such situation is realized in single mode solid laser systems with low-doped crystals $(Al_2O_3 : Cr^{3+})$ and glasses (soda-lime glass), some gas lasers (CO_2), fiber and semi-conductor lasers [3, 4, 25].

Assuming conditions $\gamma_\perp \gg \varkappa, \gamma_\parallel$, one can use the adiabatic elimination procedure which yields the relation $P = ES$. As a result, instead of the system (2) we obtain a two-component model in the form

$$\begin{cases} \dot{E} = -E(1 - S), \\ \dot{S} = \varepsilon^{-1} \left[S_e - S(1 + E^2) \right]. \end{cases} \quad (3)$$

The model (3) can not show the stable oscillating regime of the electric field E, itself. It was shown experimentally and theoretically [3, 4, 7] that stable oscillations can be realized if an additional nonlinear medium is introduced into the cavity. The first way to get the periodic lasing is to use a passive modulating medium (nonlinear material) to absorb a weak radiation and transmit signal with large amplitude. Such a type of absorbers is realized in practice as phthalocyanine fluid in Fabry–Perot cavities [26], gases SF_6, $BaCl_3$ and CO_2 [27, 28]. To describe action of the absorber it was proposed to introduce a nonlinear damping into evolution equation for the electric field [29]

$$f_\kappa = -\frac{\kappa E}{1 + E^2/E_s^2}, \quad (4)$$

here E_s is the saturation amplitude. The second way is to use an additional medium with nonlinear refractive exponent $n = n(E)$ [5, 7–9]. Such type of modulator can be used to increase the Q-factor of laser. We will model action of such an effective medium by the external force $f_e(E)$ assumed in the form

$$f_e = -A - CE^2, \quad (5)$$

that correspond to action of a bare potential $V = AE + CE^3/3$, where coefficients A, C controls photon processes in the modulator. We use an general construction (5) in order to investigate an influence of parameters A and C on lasing. In physical applications one can associate $-A$ as incident field amplitude, C can

control nonlinear properties of the refractive index $n(E)$. One of the simplest situations is considered in [19], where only a case of $A < 0$ was investigated.

Combining all above suppositions into the one model for a single-mode laser system, we will get the generalized system of nonlinear equations type of

$$\begin{cases} \dot{E} = -E(1 - S) + f_e(E) + f_\kappa(E), \\ \dot{S} = \varepsilon^{-1} \left[S_e - S(1 + E^2) \right]. \end{cases} \quad (6)$$

Using two type of additional medium in the cavity, one can expect that some combinations of parameters for both modulator and absorber should exist to provide the stable periodic radiation of the laser.

III. MAIN EQUATIONS

To find mechanisms which takes care of the stable dissipative structures formation we will use the standard procedure to analyze conditions where bifurcation into limit cycle occurs [30]. To this end we rewrite the system (6) in a most general form

$$\begin{cases} \dot{E} = f^{(1)}(E, S), \\ \varepsilon \dot{S} = f^{(2)}(E, S), \end{cases} \quad (7)$$

where effective forces are as follows:

$$f^{(1)}(E, S) \equiv -\left[A + CE^2 \right] - E - \frac{\kappa E}{1 + E^2/E_s^2} + ES,$$

$$f^{(2)}(E, S) \equiv \varepsilon^{-1} \left[S_e - S(1 + E^2) \right], \quad (8)$$

here constructions (4), (5) are used.

We deal with a problem of nonlinear dynamics and present a behaviour of the system in the phase plane (E, S). Firstly, we consider steady states E_0 and S_0, defined as coordinates of fixed points in the phase plane. Setting $\dot{E} = 0$ and $\dot{S} = 0$, one can find steady states as solutions of stationary equations

$$E_0 \left(\frac{S_e}{1 + E_0^2} - \frac{\kappa E_s^2}{E_0^2 + E_s^2} - 2CE_0 - 1 \right) = A,$$

$$S_0 = S_e(1 + E_0^2)^{-1}. \quad (9)$$

A behaviour of phase trajectories in the vicinity of these fixed points can be analyzed with a help of the Lyapunov exponents approach. Here time dependent solutions of above system are assumed to be in the form $E \propto e^{\Lambda t}, \quad \Lambda = \lambda + i\omega$, where λ controls the stability of the phase trajectories, ω determines pulse frequency of the signal. Magnitudes for real and imaginary parts of Λ are calculated according to the Jacobi matrix elements

$$M_{ij} \equiv \left(\frac{\partial f^{(i)}}{\partial x_j} \right)_{x_j = x_{j0}}; \quad x_j \equiv \{E, S\}, \quad i, j = 1, \tag{10}$$

where subscript 0 relates to steady states. Inserting (8) into definition (10), we get matrix elements

$$M_{11} = -M_0 + S_0, \tag{11}$$

$$M_0 = 1 + 2CE_0 + \kappa \frac{1 - E_0^2/E_s^2}{(1 + E_0^2/E_s^2)^2},$$

$$M_{12} = E_0; \ M_{21} = -2\varepsilon^{-1} S_0 E_0; \tag{12}$$
$$M_{22} = -\varepsilon^{-1}(1 + E_0^2).$$

Then, an equation for eigenvalues and eigenvectors

$$\sum_j M_{ij} V_j = \Lambda V_i \tag{13}$$

gives expressions for λ and ω_0 as follows:

$$\lambda = \frac{1}{2} \left[(S_0 - M_0) - \varepsilon^{-1} \left(1 + E_0^2\right) \right],$$

$$\omega_0 = \frac{1}{2} \sqrt{8\varepsilon^{-1} S_0 E_0^2 - [(S_0 - M_0) + \varepsilon^{-1}(1 + E_0^2)]^2}. \tag{14}$$

If the real part of the Lyapunov exponent $\lambda = 0$ then a fixed point (E_0, S_0) is addressed to a center of a limit cycle. It leads to relation

$$\varepsilon(S_0 - M_0) \geq 1 + E_0^2; \tag{15}$$

and yields a condition for the frequency of oscillations

$$8\varepsilon S_0 E_0^2 \geq \left[\varepsilon(S_0 - M_0) + (1 + E_0^2) \right]^2. \tag{16}$$

To investigate a stability of such a limit cycle we analyze a behaviour of trajectories in the vicinity of the fixed point (E_0, S_0). To this end we rewrite motion equations (7) where variables E and S are count off from stationary magnitudes E_0, S_0. To do this one can use following transformation

$$\vec{X} = \vec{X}_0 + \hat{P} \cdot \vec{\delta}, \tag{17}$$

where notations for pseudovectors are used:

$$\vec{X} \equiv \begin{pmatrix} E \\ S \end{pmatrix}, \quad \vec{\delta} \equiv \begin{pmatrix} E - E_0 \\ S - S_0 \end{pmatrix}. \tag{18}$$

The corresponding transformation matrix $\hat{P}$ is obtained with a help of eigenvector $\vec{V}$ components, i.e.:

$$P \equiv \begin{pmatrix} \Re V_1 & -\Im V_1 \\ \Re V_2 & -\Im V_2 \end{pmatrix}, \quad \vec{V} \equiv \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}. \tag{19}$$

Assuming $V_1 \equiv 1$, for the second component V_2 from Eq.(13) one gets

$$V_2 = \frac{(M_0 - S_0) + i\omega_c}{E_0},$$

$$\omega_c \equiv \omega_0|_{\lambda=0} = \varepsilon^{-1}(1 + E_0^2) \left[\frac{2S_e E_0^2}{(1 + E_0^2)^3} \varepsilon - 1 \right]^{1/2}. \tag{20}$$

Hence, the transformation matrix (19) takes the form

$$P = \begin{pmatrix} 1 & 0 \\ (M_0 - S_0)/E_0 & -\omega_c/E_0 \end{pmatrix}. \tag{21}$$

It leads to evolution equations for deviations written in a vector form

$$\dot{\vec{\delta}} = \vec{F}, \qquad \vec{F} \equiv P^{-1} \vec{f}. \tag{22}$$

Here a pseudovector of the canonical force

$$\vec{F} = \begin{pmatrix} F^{(1)} \\ F^{(2)} \end{pmatrix} \equiv \begin{pmatrix} f^{(1)} - f_0^{(1)} \\ f^{(2)} - f_0^{(2)} \end{pmatrix}, \tag{23}$$

satisfies conditions [30–33]

$$\frac{\partial \vec{F}}{\partial \vec{\delta}} = \begin{pmatrix} 0 & -\omega_c \\ \omega_c & 0 \end{pmatrix}, \tag{24}$$

and has following components:

$$F^{(1)} = f^{(1)}, \quad F^{(2)} = \alpha f^{(1)} - \beta \varepsilon f^{(2)}; \quad (25)$$

$$\alpha \equiv \frac{M_0 - S_0}{\omega_c}, \qquad \beta \equiv \frac{E_0}{\varepsilon \omega_c}. \quad (26)$$

Above procedure allows to find the stability of the manifold formed by the fixed point (E_0, S_0). Using the standard technique [30], one can say that the limit cycle is stable only if a real part of the Floquet exponent

$$\Phi = \frac{i}{2\omega_0} \left(g_{11}g_{20} - 2|g_{11}|^2 - \frac{1}{3}|g_{02}|^2 \right) + \frac{1}{2}g_{21}, \quad (27)$$

is negative in a bifurcation point. Structure constants in the definition (27) are described by derivatives with respect to E and S, denoted with subscripts:

$$g_{11} = \frac{1}{4} \left[\left(F_{EE}^{(1)} + F_{SS}^{(1)} \right) + i \left(F_{EE}^{(2)} + F_{SS}^{(2)} \right) \right], \quad (28)$$

$$\begin{pmatrix} g_{02} \\ g_{20} \end{pmatrix} = \frac{1}{4} \left[\left(F_{EE}^{(1)} - F_{SS}^{(1)} \mp 2F_{ES}^{(2)} \right) + i \left(F_{EE}^{(2)} - F_{SS}^{(2)} \pm 2F_{ES}^{(1)} \right) \right], \quad (29)$$

$$g_{21} = \frac{1}{8} \left\{ \left[\left(F_{EEE}^{(1)} + F_{ESS}^{(1)} \right) + \left(F_{EES}^{(2)} + F_{SSS}^{(2)} \right) \right] + i \left[\left(F_{EEE}^{(2)} + F_{ESS}^{(2)} \right) - \left(F_{EES}^{(1)} + F_{SSS}^{(1)} \right) \right] \right\}. \quad (30)$$

Using some algebra, the stability condition for the limit cycle can be written as follows

$$2\alpha(\psi_\kappa - C)^2 + \alpha\beta\varepsilon S_0(1 + 2\beta\varepsilon E_0) + \omega_c(\phi_\kappa + \beta\varepsilon) \leq$$
$$(C - \psi_\kappa)(\alpha^2 - 1 + 2\beta\varepsilon S_0 + 2\alpha\beta\varepsilon E_0), \quad (31)$$

where notations

$$\psi_\kappa = -2\frac{kE_s^2 E \left(-3E_s^2 + E^2\right)}{\left(E_s^2 + E^2\right)^3},$$

$$\phi_\kappa = 6\frac{kE_s^2 \left(-6E^2 E_s^2 + E^4 + E_s^4\right)}{\left(E_s^2 + E^2\right)^4}$$

are used.

IV. ANALYSIS OF HOPF BIFURCATIONS

A. Influence of nonlinear absorber

To proceed let us consider steady states behaviour under supposition that action of the absorber is given by expression (4), $f_e = 0$. Setting $\dot{E} = \dot{S} = 0$, one gets stationary values of the electric field amplitude E_0 shown in Fig.1. A steady states analysis allows to find that a bistable regime is realized only if $\kappa < \kappa_{min}$, here $\kappa_{min} = E_s^2/(1-E_s^2)$. In such a case one gets the hysteresis loop in $E_0(S_e)$ dependence in the domain $[S_{c0}, S_c]$ (curve 1) which disappears when the threshold κ_{min} is crossed, where

$$S_c = 1 + \kappa, \quad S_{c0} = 1 + E_s\sqrt{\frac{\kappa}{1 - E_s^2}}. \quad (32)$$

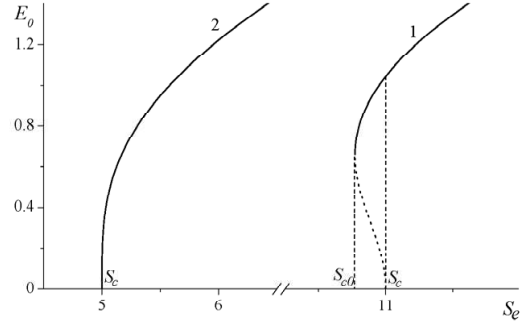The behaviour of the amplitude E_0 is the same

FIG. 1: Stationary amplitude E_0 vs. pump intensity S_e at different values of absorption coefficient at $E_s = 0.9$: curve $1 - \kappa = 10.0$; curve $2 - \kappa = 4.0$

as in the first order phase transitions where zero value of E_0 below S_{c0} corresponds to a disordered state, values $E_0 \neq 0$ (solid line) relate to an ordered state, whereas intermediate magnitudes of E_0 (dotted line) correspond to unstable state. The critical value for the absorption coefficient is realized only if the saturation amplitude $E_s < 1$. In opposite case one can get the stationary picture of the second order phase transition where E_0 increases monotonically from 0 if the critical value S_c is crossed (curve 2).

The analysis of the Floquet exponent allows to find the phase diagram (Fig.2), which shows the stable periodic radiation (formation of limit cycle in the phase plane (E, S)). In Fig.2 the do-
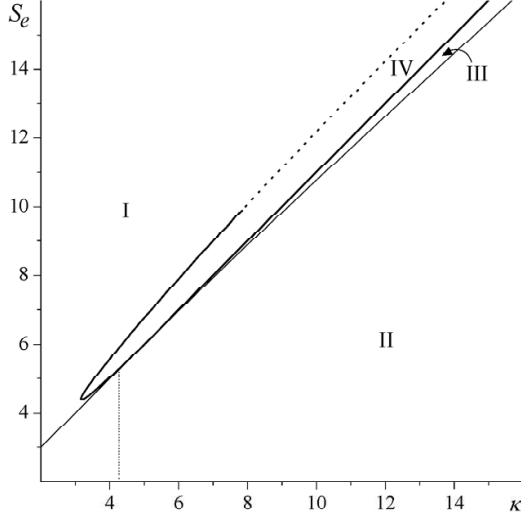

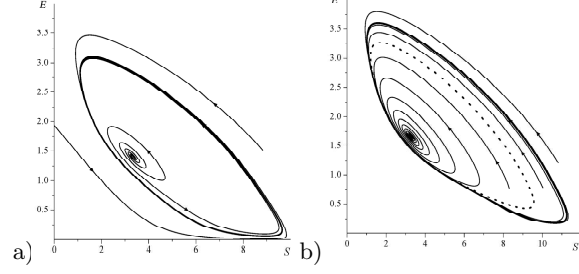FIG. 2: Phase diagram in the presence of the nonlinear absorber at $E_s = 0.9$

FIG. 3: Dissipative structures corresponding to domain IV (a) and dotted line (b) in Fig.2. Stable and unstable limit cycles: (a) — $E_s = 0.9$, $\kappa = 8.0$, $S_e = 9.9$; (b) $E_s = 0.9$, $\kappa = 10.0$, $S_e = 12.3$
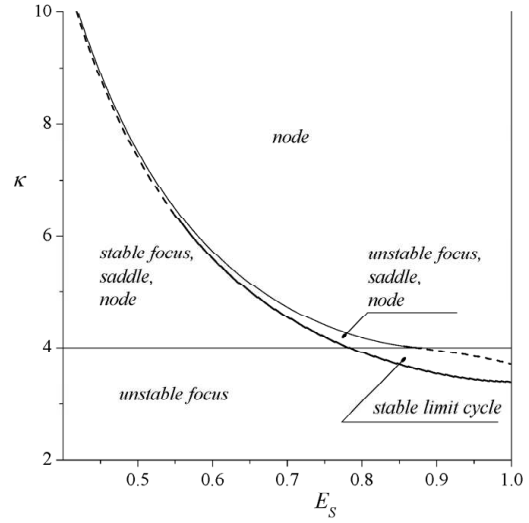

FIG. 4: Transformation of topology phase plane under influence of the absorber parameters κ, E_s at $S_e = 5.0$

main I defines configuration of the phase space with both a stable focus (ordered state) and a saddle point (disordered state); in the domain II only disordered state is realized (node); the domain III is characterized by the hysteresis loop, where ordered state corresponds to unstable focus, unstable state is represented by a saddle, disordered state is a node. Inside the domain IV the stable limit cycle is formed (Fig.3a), which transforms into stable focus, unstable and stable cycles if dotted line is crossed (Fig.3b). An influence of the parameters of the absorber on a topology of phase plane is shown in Fig.4. Here an increase in the absorption coefficient κ at small E_s leads to transformation of unstable focus into a stable one with additional node and saddle points appearing. At values κ and E_s, corresponding to the dashed line, one gets an unstable limit cycle and in the domain bounded by dashed and solid lines one gets the unstable focus, node and saddle. When the solid line is crossed the phase portrait is characterized by

a single node. An increase in κ at saturation amplitude $E_s \simeq 1$ transforms an unstable focus into a stable limit cycle, which becomes unstable at values that correspond to the dashed line. In the domain bounded by the dashed and straight horizontal lines there is a single unstable focus only. A further increase in κ transforms this focus into a node.

The frequency of pulse radiation regime appears at non zero value, that correspond to the first bifurcation point S_e a further increase in

the pump intensity, leads to the growth of ω_c till the second critical point S_e is achieved. We have analyzed behavior of pulse radiation frequency at different values of the absorption coefficient κ. According to Fig.5 an increase in κ at fixed saturation amplitude magnitudes leads to the shift of minimal and maximal values of ω_c despite a topology of the dependence $\omega_c(S_e)$ is not changed. Obtained results are in good corresponding with experimental observations of such dependence [34].
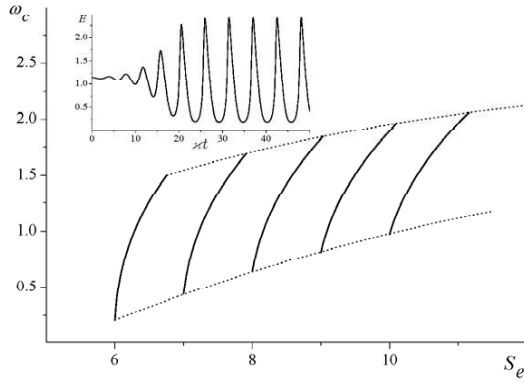
FIG. 5: Dependence of the stable pulse radiation frequency ω_c vs. pump intensity S_e for the system with absorption effect at $E_s = 0.9$ and $\kappa = 5, 6, 7, 8, 9$ (from the left to the right). Insertion shows a typical stable pulse signal

Therefore, the dispersion in the relaxation time of the electric field amplitude E, promoting by the absorbing influence, leads to formation of the stable periodic radiation at saturation amplitude $E_s \simeq 1$.

B. Influence of external modulator

Let us consider an influence of the external source f_e at $f_\kappa = 0$. It is principally important that the periodic radiation is possible only if parameter that controls nonlinear effects $C < 0$. Here stationary behavior of the field E_0 versus pump intensity S_e is shown in Fig.6. Analysis of the Floquet exponent shows that limit cycles
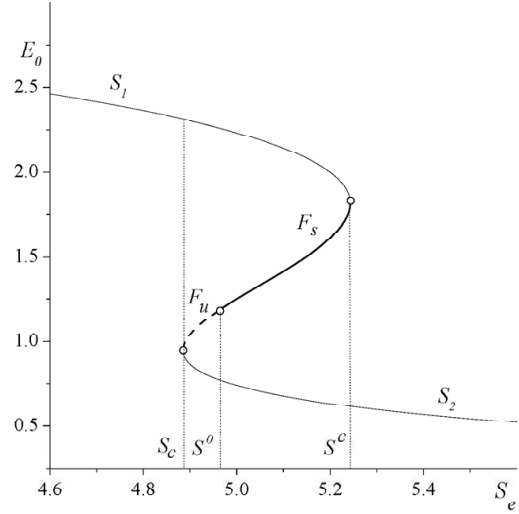

FIG. 6: Stationary amplitude E_0 vs. S_e at $A = 1.9$, $C = -0.455$

can be formed only if a stable focus is transformed into an unstable one and vice versa (see Fig.6). Here at $S_e < S_c$ and $S_e > S^c$ the phase portrait is characterized by single saddle point S_1 or S_2, respectively. In the domain $S_c < S_e < S^0$ one gets two saddles S_1 and S_2, divided by an unstable focus F_u. If $S^0 < S_e < S^c$, then such saddles are divided by a stable focus F_s. Only if $S_e = S^0$ we will get a trivial situation, where $\Re\Phi = 0$. It means a formation of nested loops of neutral stability (Fig.7). Therefore, external force suppresses processes of dissipative structure formation.

C. Combined effect of external modulator and nonlinear absorber

Now we consider an influence of both external modulator and nonlinear absorber on the processes of dissipative structure formation. Setting $\dot{E} = \dot{S} = 0$ in the system (7), one gets stationary values of the electric field amplitude E_0 shown in Fig.8. As it is seen, if the modulator is turned off ($A = C = 0$) then we have a single stable state with no radiation at small values of the pump parameter S_e. If the thresh-
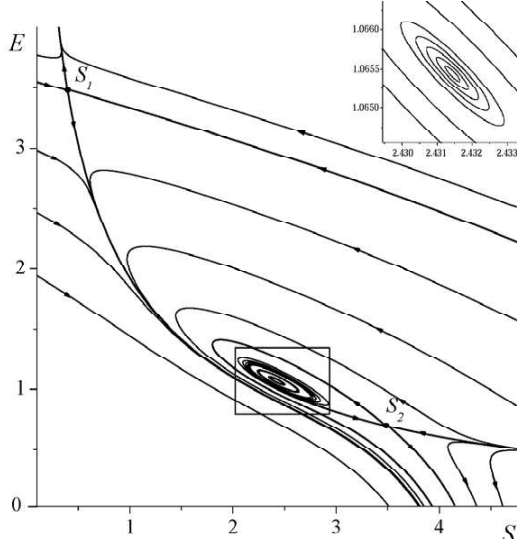
FIG. 7: Typical phase portrait of the system with external source at $S_e = 5.19152$ $A = 1.9$, $C = -0.33025$
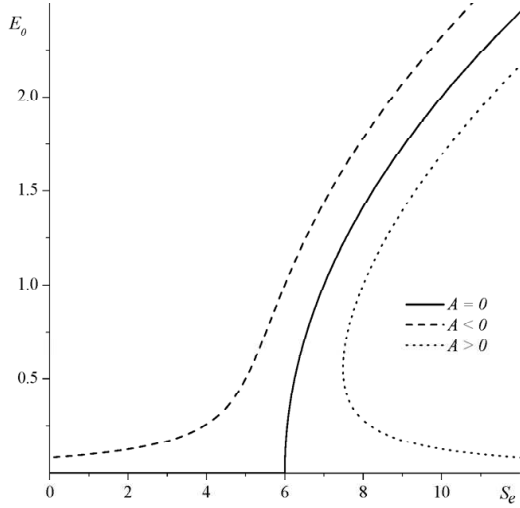

FIG. 8: Stationary value of electric field amplitude E_0 vs. pumping intensity S_e for the system (7) at $C = 0$, $\kappa = 5.0$ and different magnitudes for the parameter of spontaneous emission A

old given by expression $S_e^c = 1 + \kappa$ is crossed,

then a new solution of the steady state equation appears and we have a stationary radiation with an amplitude $E_0 \neq 0$ which increases with an increase in the pump intensity. If we set $A < 0$ at $C = 0$ then we will get a single stable solution on the whole axis of the pump parameter magnitudes which defines the radiation amplitude E_0. In the opposite case of $A > 0$ one gets two stationary solutions, only if the energy barrier S_e^c, given by the solution of equation $S_e^c = f(A, \kappa_0, \kappa)$, is overcame.

Next, we investigate conditions where stable periodic radiation can be realized. To this end we need to determine a domain defined by conditions $\lambda = 0$ and $\Re\Phi < 0$ where periodic solutions of the system (7) are exist. Corresponding solutions of the Eq.(31) are shown in Fig.9. It illustrates domains of the absorption coefficient κ and pump intensity S_e magnitudes at different intensities A, C where the stable radiation process is realized. As Fig.9 shows, if we set an absorber inside the cavity only, then a semi-limited domain of κ and S_e magnitudes is formed; inside of this domain the stable periodic radiation is possible. Introducing a modulator with $A > 0$, $C = 0$ (see Fig.9a), such a domain becomes totally limited. Moreover, an increase in the parameter A leads to restriction of the values for the collective parameter κ and pump intensity S_e, at which one has stable periodic radiation. At large values A such domain is degenerated into the line. From Fig.9b one can see that an increase in the C at $A = 0$ leads to extension of the domain of stable periodic radiation that occurs at large magnitudes of pump intensity parameter.

An influence of nonlinear processes in the modulator on a picture of the stable periodic radiation formation is presented in Fig.10. It is seen, if $A < 0$ then there is only stable stationary state (see Fig.8) which is a focus ($\Re\Lambda < 0$, $\Im\Lambda \neq 0$) on a phase plane (E, S). Such a fixed point is transformed into a manifold if control parameters are in the domain including its border shown in Fig.10a. Such a manifold is a limit cycle ($\Re\Phi < 0$, $\Re\Lambda = 0$) in the phase plane (E, S), that attracts all phase trajectories in the vicinity of it. From a physical viewpoint
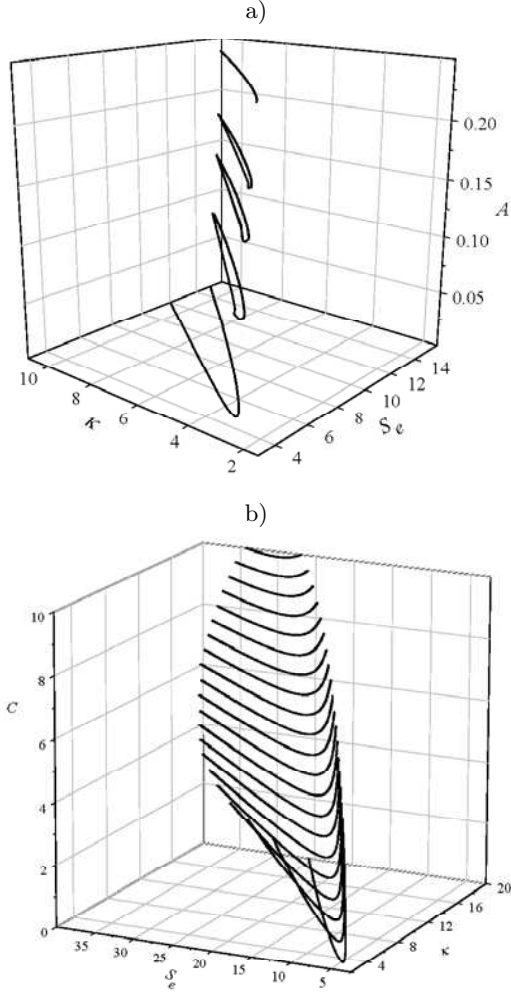
a)

b)

FIG. 9: Phase diagrams of periodic radiation effect: (a) an influence of the spontaneous emission intensity A at $C = 0$; (b) influence of the photon scattering intensity C at $A = 0$
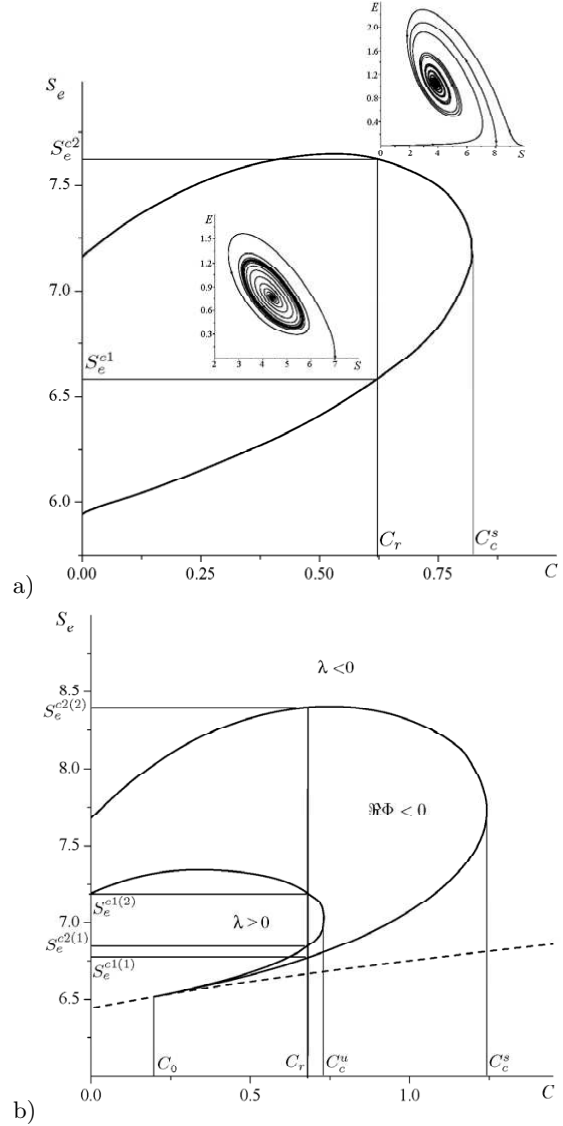
a)

b)

FIG. 10: Phase diagrams of Hopf bifurcation: influence of the nonlinear processes intensity C at $\kappa = 5.0$: (a) $A = -0.1$; (b) $A = 0.1$

it means the formation of the stable pulse periodic radiation. The domain shown in Fig.10a is limited by the value of intensity of nonlinear processes C_c^s. One needs to note that if photon scattering occurs with intensities $C < C_c^s$ then an increase in pump intensity S_e induces formation of stable periodic radiation at magnitude S_e^{c1} and destroys it at S_e^{c2}. In other words,

one gets the situation where the only one reason serves as stimulus for both self-organization and desorganization.

A picture became more complicated at $A > 0$. At first let us discuss the phase diagram shown in Fig.10b. At pump limited by the dashed

curve in Fig.10b there are no stationary solutions and, hence, no stable regimes of radiation. Next, processes of spontaneous photon annihilation reduce a domain of stable periodic radiation at pump intensities above dashed curve, here a domain of unstable behaviour of phase trajectories appears. At small $C < C_0$ such a stationary regime is defined by the corresponding stationary solution which is an unstable focus ($\Re\Lambda > 0$, $\Im\Lambda \neq 0$). The related fixed point is defined as an upper branch of the dashed curve in Fig.8. At large values $C_c^u < C < C_c^s$ one has a picture similar to discussed above. At intermediate values $C_0 < C_r < C_c^u$ one can get a very complicated picture of self-organization. Here with an increase in S_e we have following picture of transformations: (i) the system passes from the unstationary regime to stationary one with fixed point to be a stable focus (Fig.11a); (ii) at values $S_e^{c1(1)}$ one has a stable periodic radiation that exists till magnitudes $S_e < S_e^{c2(1)}$ (Fig.11b); (iii) a further increase in pump intensity destroys the limit cycle and a system pass to unstable regime which is characterized by the unstable focus (Fig.11c); (iv) if critical value $S_e^{c1(2)}$ is achieved, then a new Hopf bifurcation occurs and the system evolves according to periodic trajectories (Fig.11b); (v) at last, such a coherent regime is destroyed at $S_e > S_e^{c2(2)}$ (Fig.11a).

Let us consider more closely properties of phase diagram (Fig.4), which shows magnitudes of the absorption coefficient κ and the control parameter S_e. Here the thin solid curve (bifurcation line) defines critical magnitudes for κ and S_e where stationary states appeared. In the domain with $\Re\Lambda > 0$ one has an unstable focus (see Fig.13a). The stable limit cycle is realized inside the bounded domain with $\Re\Phi < 0$ (Fig.13b). At small κ and large S_e one has a stable focus (Fig.13c). Therefore, one gets a transformation of topology of attractors in the phase plane if parameters S_e or κ are changed. An increase in absorption coefficient κ at fixed pump intensity S_e will produce the oscillating regime (transition from a stable focus to limit cycle). Such stable periodic regime can be destroyed at

large magnitudes of κ (transition from the limit cycle into repeller — unstable focus) and a further increase in κ leads to the absence of any stationary regime at all. However, the stable periodic solution is observed not on a whole border of the indicated domain. Figure 12 shows that a stable dissipative structure is formed inside the domain and on the thick solid lines only ($\Re\Phi < 0$). A part of the domain border plotted as dashed line corresponds to conditions $\lambda = 0$ and $\Re\Phi > 0$, which mean existence of unstable periodic solution (see Fig.13d). Hence, there is a point where $\Re\Phi = 0$ and periodic solution changes its stability. In this point the phase portrait of the system is characterized by a set of nested loops.

V. CONCLUSIONS

In this Paper we have analyzed properties of self-organization processes in the two-level class-B laser systems in the presence of absorption effects and influence of the external force. We have shown that due to the nonlinear damping the domain of control parameters of the cavity with the stable pulse radiation is realized. It was shown that varying a saturation amplitude and absorption coefficient one can pass to different type of radiation, characterized by fixed point in the phase space type of: stable and unstable focuses, stable and unstable limit cycles. Introducing the external force that leads to additional nonlinear effects that reduce domains of control parameters with stable periodic radiation. It is principally important that due to the external force influence one can get reentrant Hopf bifurcation. Here there is a wide range of the external force parameters, where both stable and unstable dissipative structures are in the phase space. Our results are in good correspondence with theoretical ones [3, 17] and experimental observations [4, 27, 28, 34, 35].

In our investigation we have considered the simplest case, where relaxation velocities of the electric field and population inversion are of the same order. In real systems of the solid-state class-B lasers $\varepsilon \equiv \varkappa/\gamma_\parallel \sim 10^{-1} \div 10^{-3}$, in

gas lasers of such class $\varepsilon \simeq 1$. As was shown theoretically and experimentally [27] a difference between above relaxation velocities will not change the picture of stable pulse regime qualitatively. Experimental investigation shows quantitative changes only.

In our consideration the construction for the external force can be applied to describe influence of the nonlinear processes: in the nonlinear medium with the nonlinear dependence of refractive index (a variation of the parameter C); introducing an external incident field with amplitude $A < 0$; more complicated picture with arbitrary A and C under supposition of the dynamic system stability only.

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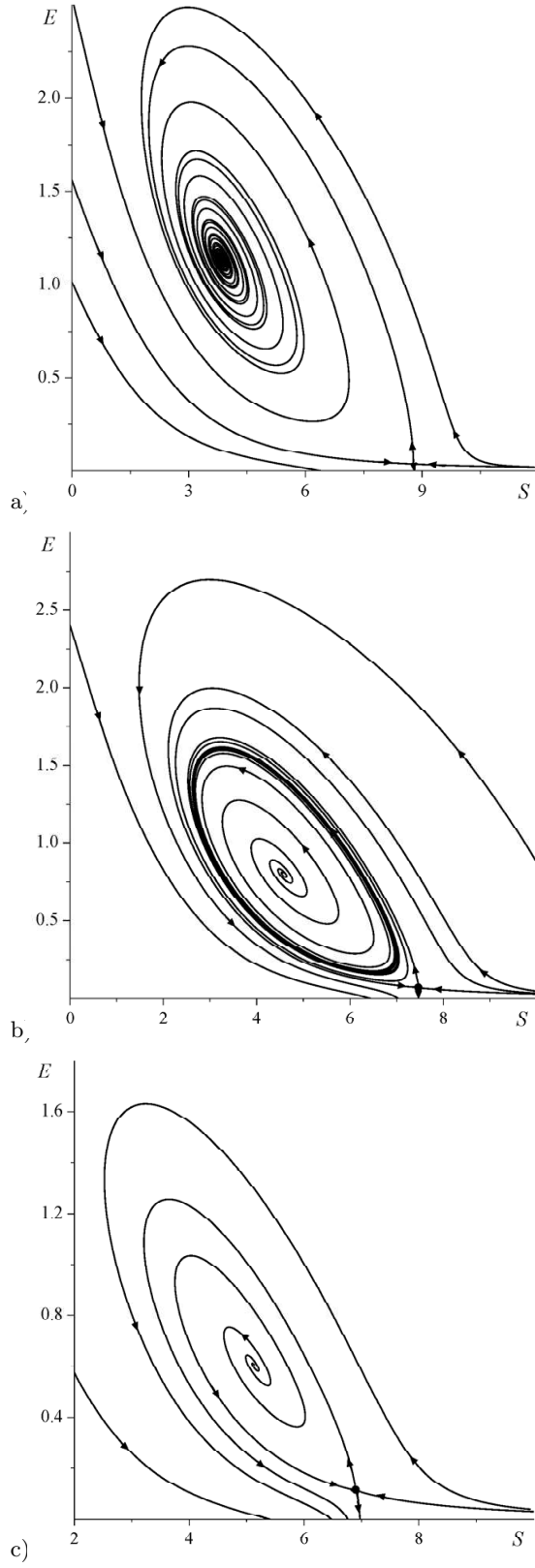[35] F.V.Garcia-Ferrer, I.Perez-Arjona, G.J. de Valcarcel, E.Roldan, quant-ph/0702113.

FIG. 11: Phase portraits related to the domains in
Fig.10b: (a) − $A = 0.1$, $C = 0.5$, $\kappa = 5.0$, $S_e = 8.8$;
(b) − $A = 0.1$, $C = 0.5$, $\kappa = 5.0$, $S_e = 7.5$; (c) −
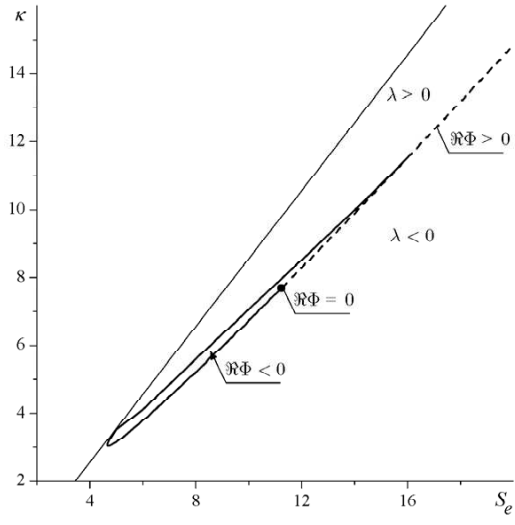$A = 0.1$, $C = 0.5$, $\kappa = 5.0$, $S_e = 7.0$

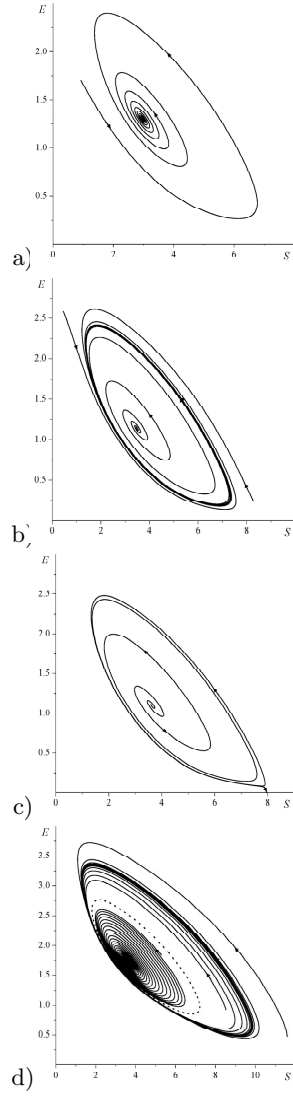FIG. 12: Phase diagram at $A > 0$, $C = 0$

FIG. 13: Phase portraits related to the phase diagram in Fig.12: (a) – $A = 0.1$, $C = 0.0$, $\kappa = 5.1$, $S_e = 8.0$; (b) – $A = 0.1$, $C = 0.0$, $\kappa = 5.5$, $S_e = 8.0$; (c) – $A = 0.1$, $C = 0.0$, $\kappa = 5.6$, $S_e = 8.0$; (d) – $A = 0.1$, $C = 0.0$, $\kappa = 9.86$, $S_e = 14.0$.